

Anyonic Bogomol'nyi Solitons in a Gauged $O(3)$ Sigma Model

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Abstract

We introduce the self-dual abelian gauged $O(3)$ sigma models where the Maxwell and Chern-Simons terms constitute the kinetic terms for the gauge field. These models have quite rich structures and various limits. Our models are found to exhibit both symmetric and broken phases of the gauge group. We discuss the pure Chern-Simons limit in some detail and study rotationally symmetric solitons.

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Recently the gauged $O(3)$ sigma models [1,2] have been studied in three dimensions with the Maxwell term where the gauge group $U(1)$ is a subgroup of $O(3)$. These models differ from the gauged $O(3)$ sigma model previously discussed [3–5] in that the gauge field is coupled to the scalar fields through a $U(1)$ electric current rather than the topological one. If a specific potential is chosen, then the Bogomol’nyi type energy bound is found to exist. This bound is shown to be saturated by topological lumps carrying nonzero magnetic flux. The size of these topological lumps depends on the magnetic flux but their mass is independent of the magnetic flux. Subsequently, the self-dual gauged sigma models with both Maxwell and Chern-Simons terms have been discussed [6], where both topological and nontopological solitons are found.

However, the vacua of all above models respect the gauge symmetry, so the solitons live in the symmetric phase. In this paper, we generalize these models such that the asymmetric phase where the gauge symmetry is spontaneously broken is also admitted. For the generality, we may also include a uniform external charge density. It seems that the structures of these models are rather rich.

The models exhibit two inequivalent symmetric phases and one asymmetric phase, depending on a parameter of the theory. By studying the rotationally symmetric self-dual solitons, we found that in the symmetric phases there are nontopological Q -lumps [7] with or without vortices at their centers. Also there are topological lumps carrying nonzero unquantized magnetic flux. In general their mass and size depend on their magnetic flux. In the broken phase, there are two types of topological vortices of quantized magnetic flux. Curiously, it seems in the asymmetric phase that there exist topological lumps which are not rotationally symmetric.

Let us start with a scalar field $\phi(x)$ which is a map from the 3 dimensional spacetime to the two-sphere of unit radius. As we have scaled ϕ to be a unit vector, the spatial coordinates and the coupling parameters will be dimensionless. For a given $\phi(x)$ field configuration, one can construct a topological current [8]

$$k_\alpha = \frac{1}{8\pi} \epsilon_{\alpha\beta\gamma} \boldsymbol{\phi} \cdot \partial^\beta \boldsymbol{\phi} \times \partial^\gamma \boldsymbol{\phi}, \quad (1)$$

which is conserved trivially. If $\boldsymbol{\phi}$ approaches a constant unit vector at the spatial infinity, we can compactify two dimensional space as a unit sphere and regard $\boldsymbol{\phi}$ as a mapping from a two-sphere to a two-sphere. The integer-valued associated degree in this case is given by the charge $S = \int d^2x k_0$ of the topological current.

We introduce the $U(1)$ gauge coupling by a covariant derivative

$$D_\alpha \boldsymbol{\phi} = \partial_\alpha \boldsymbol{\phi} + A_\alpha \mathbf{n} \times \boldsymbol{\phi}, \quad (2)$$

where $\mathbf{n} = (0, 0, 1)$ is a unit vector. The $U(1)$ gauge group is a subgroup of the $O(3)$ rotational group of $\boldsymbol{\phi}(x)$. The gauge invariant generalization of the topological current k_α is

$$K_\alpha = \frac{1}{8\pi} \epsilon_{\alpha\beta\gamma} \left(\boldsymbol{\phi} \cdot D^\beta \boldsymbol{\phi} \times D^\gamma \boldsymbol{\phi} + F^{\beta\gamma} (v - \mathbf{n} \cdot \boldsymbol{\phi}) \right), \quad (3)$$

where v is a free real parameter. This current differs from k_α only by the curl of a vector field:

$$K_\alpha = k_\alpha + \frac{1}{4\pi} \epsilon_{\alpha\beta\gamma} \partial^\beta ((v - \mathbf{n} \cdot \boldsymbol{\phi}) A^\gamma). \quad (4)$$

The corresponding conserved topological charge is $T = \int d^2x K^0$ which may differ from the degree S .

As we will see, we can impose various boundary conditions at spatial infinity on the finite energy configurations. Depending upon the boundary conditions, the gauge symmetry can be realized in the symmetric phase or the asymmetric one: (1) For the symmetric phases

$$\lim_{|\mathbf{x}| \rightarrow \infty} \boldsymbol{\phi}(t, \mathbf{x}) = \pm \mathbf{n}, \quad (5)$$

where the gauge boson is massless; (2) If $|v| < 1$, one can impose

$$\lim_{|\mathbf{x}| \rightarrow \infty} \mathbf{n} \cdot \boldsymbol{\phi}(t, \mathbf{x}) = v. \quad (6)$$

and the asymmetric phase is realized. In the symmetric phase, the mapping $\boldsymbol{\phi} : R^2 \rightarrow S^2$ can be regarded as a map from a two-sphere to another two-sphere, and S measures the

associated second homotopy which takes an integer value. However, in the asymmetric phase, the mapping ϕ is not necessarily such a map. The vacuum manifold is a unit circle and $\phi(t, r = \infty, \theta)$ in the polar coordinate (r, θ) of the spatial plane is a map from $S^1 \rightarrow S^1$. If the mapping at the spatial infinity has any nontrivial first homotopy, the mapping $\phi : R^2 \rightarrow S^2$ covers the two-sphere partially and the degree S becomes fractional. On the other hand, if the mapping at the spatial infinity has the trivial homotopy, the degree S will be an integer.

In general the kinetic term for the gauge field in three dimensions consists of the Maxwell term and the Chern-Simons term and a uniform external charge density may be coupled to the gauge field [9,10]. In order to make the model self-dual, we also need a neutral scalar field N which couples to the ϕ field. Hence the general Lagrangian we consider here is given by

$$\mathcal{L}_1 = -\frac{1}{4e^2}(F_{\alpha\beta})^2 + \frac{\kappa}{2}\epsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma + \frac{1}{2e^2}(\partial_\alpha N)^2 + \frac{1}{2}(D_\alpha\phi)^2 - U(\phi, N) - \rho_e A_0, \quad (7)$$

where e^2, κ, ρ_e are dimensionless parameters. By a similar procedure as in Ref. [11] we can find the potential U with which the Bogomol'nyi type bound on the energy functional exists. The potential are found to be one of two special potentials:

$$U(\phi, N)_\pm = \frac{e^2}{2}(\kappa N + (v - \mathbf{n} \cdot \phi))^2 + \frac{1}{2}N^2(\mathbf{n} \times \phi)^2 \mp \rho_e N. \quad (8)$$

Then, the energy functional is bounded by $\pm 4\pi T$ depending on the sign of the potential.

When $\rho_e \neq 0$, this self-dual model seems to have very rich structures similar to those considered in Ref. [10] and we hope to pursue them elsewhere. Here we restrict ourselves to the case $\rho_e = 0$ for the sake of simplicity. In the pure Maxwell limit $\kappa \rightarrow 0$, the potential becomes

$$U_m = \frac{e^2}{2}(v - \mathbf{n} \cdot \phi)^2 + \frac{1}{2}N^2(\mathbf{n} \times \phi)^2 \quad (9)$$

The model considered in Ref. [1] corresponds to case when $v = 1$. On the other hand, in the pure Chern-Simons limit, $e^2 \rightarrow \infty$, classically $N = -(v - \mathbf{n} \cdot \phi)/\kappa$ and the potential becomes

$$U_{cs} = \frac{1}{2\kappa^2}(v - \mathbf{n} \cdot \phi)^2(\mathbf{n} \times \phi)^2. \quad (10)$$

This model with $v = 1$ is the one considered in Ref. [6].

In this paper we focus our attention on the pure Chern-Simons case where the Lagrangian is given by

$$\mathcal{L}_{cs} = \frac{\kappa}{2} \epsilon^{\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma + \frac{1}{2} (D_\alpha \phi)^2 - \frac{1}{2\kappa^2} (v - \mathbf{n} \cdot \phi)^2 (\mathbf{n} \times \phi)^2. \quad (11)$$

When $|\phi - \mathbf{n}| \ll 1$ and $0 < 1 - v \ll 1$, the above model reduces to the self-dual Chern-Simons Higgs systems [12,13]. When $|v| < 1$, there are three degenerate minima: (1) the two symmetric phases where $\phi = \pm \mathbf{n}$ and the mass of charged scalar particles is $m_\pm = |(v \pm 1)/\kappa|$. (2) the broken phase where $\mathbf{n} \cdot \phi = v$ and the masses of neutral vector and scalar particles are given by $|(1 - v^2)/\kappa|$. When $|v| \geq 1$, only the symmetric phases remain.

The Gauss law obtained from the variation of A_0 is

$$\kappa F_{12} + \mathbf{n} \cdot \phi \times D_0 \phi = 0, \quad (12)$$

and the gauged $U(1)$ current is given by $J^\alpha = \mathbf{n} \cdot \phi \times D^\alpha \phi$. The Gauss law implies that the total magnetic flux $\Psi = \int d^2x F_{12}$ is related to the total charge $Q = \int d^2x J^0$ by the equation $\kappa \Psi = -Q$.

The conserved energy functional is

$$E = \frac{1}{2} \int d^2x \left((D_0 \phi)^2 + (D_i \phi)^2 + U_{cs} \right). \quad (13)$$

The energy functional can be rewritten as

$$E = \frac{1}{2} \int d^2x \left((D_0 \phi \pm \frac{1}{\kappa} (v - \mathbf{n} \cdot \phi) \mathbf{n} \times \phi)^2 + (D_1 \phi \pm \phi \times D_2 \phi)^2 \right) \pm 4\pi T, \quad (14)$$

where we take advantage of $\phi \cdot D_\alpha \phi = 0$ and the Gauss law. Here we also discarded a total derivative whose contribution vanishes. Thus, there is a bound on the energy,

$$E \geq 4\pi |T|, \quad (15)$$

and this bound is saturated if the self-dual equations hold:

$$D_1 \phi = \mp \phi \times D_2 \phi, \quad (16)$$

$$D_0 \phi = \pm \frac{1}{\kappa^2} (v - \mathbf{n} \cdot \phi) (\mathbf{n} \times \phi). \quad (17)$$

The second equation yields $\mathbf{n} \cdot \partial_0 \boldsymbol{\phi} = 0$, which implies the energy density is static in time. We can choose the gauge such that the field configurations satisfying the self-dual equations are static. Putting Eqs. (12) and (17) together, we find

$$F_{12} = \pm \frac{1}{\kappa^2} (v - \mathbf{n} \cdot \boldsymbol{\phi}) (\mathbf{n} \times \boldsymbol{\phi})^2. \quad (18)$$

The conserved angular momentum is given by

$$J = - \int d^2x \epsilon_{ij} x^i D_0 \boldsymbol{\phi} \cdot D_j \boldsymbol{\phi}. \quad (19)$$

As we will see, the solitons in this system carry in general fractional angular momenta, thus can be regarded as anyons.

In the parameterization

$$\boldsymbol{\phi}(x) = (\sin f(x) \cos \psi(x), \sin f(x) \sin \psi(x), \cos f(x)), \quad (20)$$

we may rewrite the conserved angular momentum as

$$J = \int d^2x \epsilon_{ij} x^i \left(\partial_0 f \partial_j f - \kappa (\partial_j \psi + A_j) F_{12} \right). \quad (21)$$

if we make use of the Gauss law. In terms of this parameterization the self-dual equations become

$$\partial_i f = \pm \epsilon_{ij} (\partial_j \psi + A_j) \sin f, \quad (22)$$

$$\kappa^2 F_{12} = \pm (v - \cos f) \sin^2 f. \quad (23)$$

From these equations one can deduce that there may be two kind of vorticities, of which asymptotics are $f = 0$ and $f = \pi$.

Let us now consider rotationally symmetric solitons satisfying the self-dual equations. In the polar coordinate (r, θ) , let us take an ansatz $f = f(r)$, $\psi = N\theta$ with an integer N and $A_\theta = a(r)$ to respect the rotational symmetry. We may choose the upper sign in the above self-dual equations without lose of generality, assuming that the degree T is positive. The self-dual equations become

$$rf'(r) = (N + a(r)) \sin f(r), \quad (24)$$

$$a'(r) = \frac{r}{\kappa^2} (v - \cos f(r)) \sin^2 f(r). \quad (25)$$

With $\alpha \equiv a(\infty)$ the degree for this ansatz is given by

$$T = \frac{N}{2} [\cos f(0) - \cos f(\infty)] + \frac{\alpha}{2} [v - \cos f(\infty)], \quad (26)$$

while the magnetic flux and the angular momentum of the solution are $\Psi = 2\pi\alpha$ and $J = \pi\kappa((N + \alpha)^2 - N^2)$.

Eqs.(24) and (25) tell the behavior of the solution near $r = 0$, which is rather straightforward. For A_i to be nonsingular at the origin, $a(0) = 0$. When $f(0) \neq 0, \pi$, ϕ is nonsingular only if $N = 0$. When $f(0) = 0$, N should be positive and $f(r) \propto r^N$ near $r = 0$. When $f(0) = \pi$, N should be negative and $f(r) \propto r^{|N|}$ near $r = 0$.

The behavior of the solution near $r = \infty$ depends on whether $f(\infty) = 0, f_v$ or π : When $f(\infty) = 0$, $f(r) \propto r^{N+\alpha}$ and $a(r) - \alpha \propto r^{2(N+\alpha+1)}$ for large r . In this case the asymptotic behavior is consistent if $\alpha < -1 - N$. When $f(\infty) = \pi$, $\pi - f(r) \propto r^{-N-\alpha}$ and $a(r) - \alpha \propto r^{-2N-2\alpha+2}$. Thus, the consistent asymptotic behavior requires that $\alpha > 1 - N$. When $f(\infty) = f_v$, $\alpha = -N$ and $f(r), a(r)$ approach to their asymptotic values exponentially.

From Eq.(24) it follows that the range of the function $f(r)$ lies between 0 and π . We can show this by the contradiction. Let us assume $f(r)$ to cross zero near $r = r_0$. From Eq.(24), we see that near $r = r_0$, $f \propto e^{(N+a(r_0))(r-r_0)/r_0}$, which cannot vanish. Thus, $f(r) > 0$. Also the same procedure shows that it cannot cross π in the range $(0, \infty)$, i.e., $f(r) < \pi$.

Now we call our attention to the solutions when $|v| < 1$ and discuss the results of the numerical analysis. Then we will consider later the cases of $|v| \geq 1$ where the solutions for $|v| < 1$ become degenerate.

a) When $f(0) \neq 0, \pi$ and $N = 0$.

We define $0 < f_v < \pi$ such that $\cos f_v = v$. From Eqs.(24) and (25) we see that if $f(0) > f_v$, both $f(r)$ and $a(r)$ are increasing functions. At spatial infinity, it should be that

$f(\infty) = \pi$ and $\alpha > 1$. The solution has the mass $2\pi(v+1)\alpha$ and the angular momentum $J = \pi\kappa\alpha^2$. This is a nontopological soliton in the symmetric phase $f = \pi$ without any vortex in its center. If $f(0) < f_v$, we see that $f(r)$ and $a(r)$ are decreasing function with $f(\infty) = 0$ and $\alpha < -1$. Its mass and spin are given by $2\pi(1-v)(-\alpha)$ and $J = \pi\kappa\alpha^2$ respectively. It is again the nontopological soliton in the symmetric phase $f = 0$ without any vortex in its center. Note that the charged particles in the symmetric phases $f = 0, \pi$ have the same mass per charge ratio as these nontopological solitons, so they are marginally stable. Fig.1 depicts $f(r)/\pi$ for the cases $f_v = \pi/3$ with two different choices for the values of $f(0)$. Fig.2 shows $a(r)$ for the same initial values.

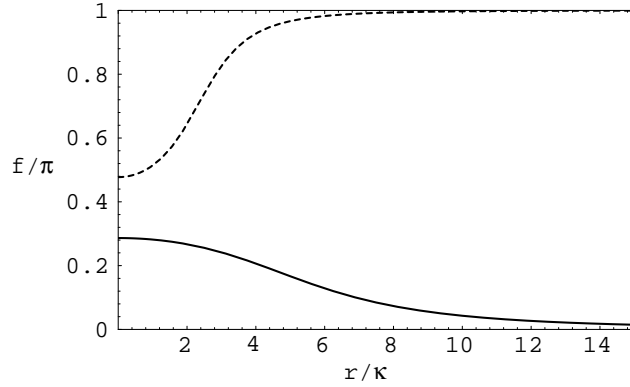


Fig.1 Plotting $f(r)$ for $N = 0$ and $f(0) \neq 0$ and $f_v = \pi/3$. The dotted line is for $f(0) = 1.5$ and the solid line for $f(0) = 0.9$.

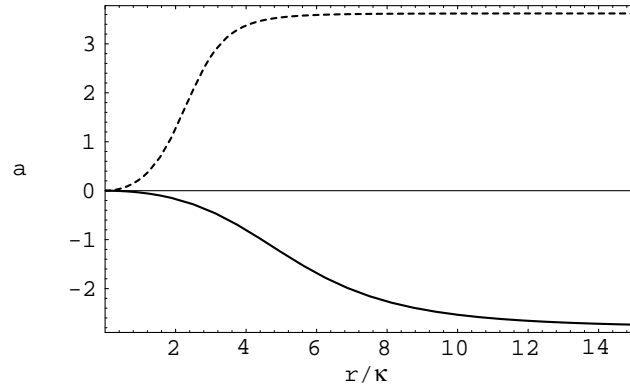


Fig.2 Plotting $a(r)$ for $N = 0$, $f(0) \neq 0$ and $f_v = \pi/3$. The dotted line is for $f(0) = 1.5$ and the solid line for $f(0) = 0.9$ respectively.

b) When $f(0) = 0$ and $N \geq 1$.

Let us now consider the case $f(0) = 0$. There are three kinds of solutions depending on whether $f(\infty) = 0, f_v$ or π . If $f(\infty) = \pi$, we see that $\alpha > -N + 1$. The function $f(r)$ is a monotonously increasing function, while $a(r)$ decreases and then increases as r increases, approaching to the value α . One can see that the minimum of $a(r)$ is larger than $-N$. Its mass and angular momentum are $4\pi N + 2\pi(v + 1)\alpha$, $J = \pi\kappa^2\alpha(\alpha + 2N)$ respectively. It is a topological lump in the symmetric phase $f = \pi$ with a nonzero degree $S = N$.

When $f(\infty) = f_v$, $f(r)$ is monotonically increasing while $a(r)$ is monotonically decreasing with $a(\infty) = -N$. It is a vortex in the asymmetric phase with magnetic flux $-2\pi N$ and mass $2\pi N(1 - v)$. Its spin is $-\pi\kappa N^2$.

When $f(\infty) = 0$, we see $a(r)$ is monotonically decreasing to $\alpha < -(N + 1)$. Here $f(r)$ increases and then decreases, approaching to zero. This is a nontopological soliton with a vortex in the middle. Fig.3 describes the function $f(r)$ for these three cases with $N = 1, f_v = \pi/3$ in terms of the value α . Fig.4 shows $a(r)$ for the same values of α .

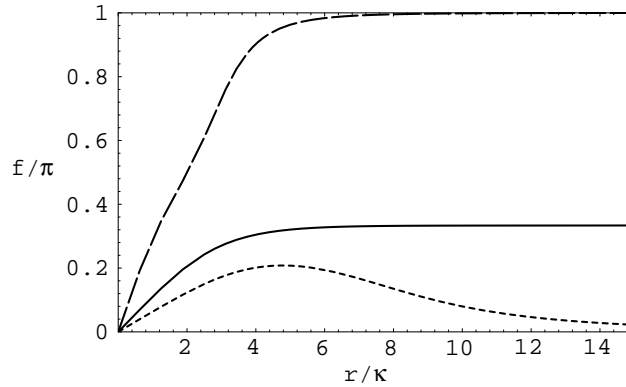


Fig.3 Plotting $f(r)$ for $N = 1$, $f(0) = 0$ and $f_v = \pi/3$. The dashed line corresponds to $\alpha = 3.53325$, the solid line to $\alpha = -1.0$ and the dotted line to $\alpha = -4.50025$ respectively.

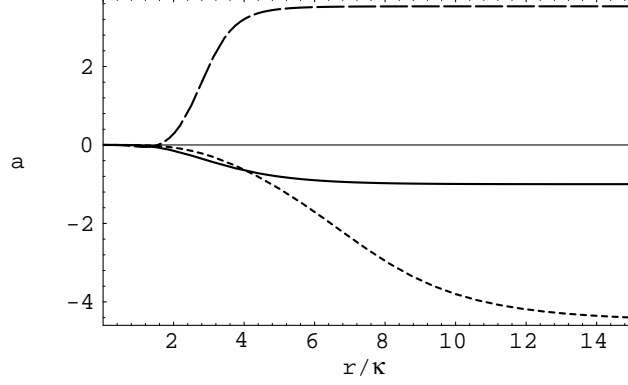


Fig.4 Plotting $a(r)$ for $N = 1$, $f(0) = 0$ and $f_v = \pi/3$. The dashed line is for $\alpha = 3.53325$, the solid line for $\alpha = -1.0$ and the dotted line for $\alpha = -4.50025$ respectively.

c) When $f(0) = \pi$ and $N \leq -1$.

The solution in this case is similar to the previous one. There are again three kinds of solutions depending on $f(\infty) = 0, f_v$ or π . The behaviors of the function $f(r)$ and $a(r)$ are quite similar to those in the previous case except their sign. Fig.5 shows the function $f(r)$ for these three cases with $N = 1, f_v = \pi/3$. Fig.6 draws $a(r)$ for these cases.

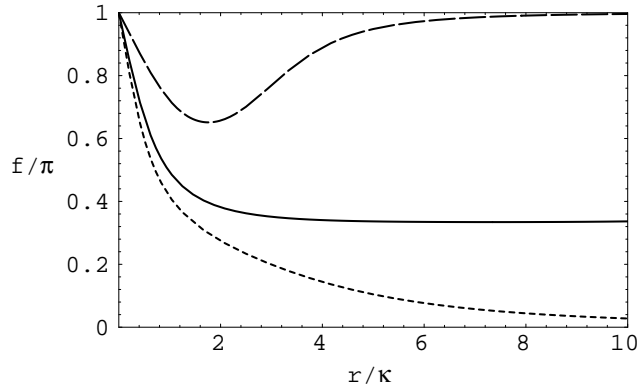


Fig.5 Plotting $f(r)$ for $N = -1$, $f(0) = \pi$ and $f_v = \pi/3$. The dashed line is for $\alpha = 4.77074$, the solid line for $\alpha = 1.0$ and the dotted line for $\alpha = -1.29324$ respectively.

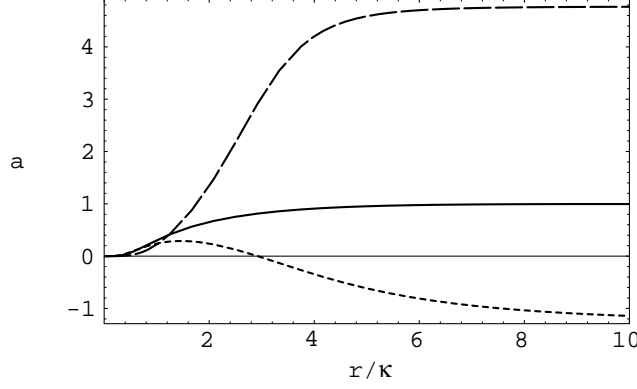


Fig.6 Plotting $a(r)$ for $N = -1$, $f(r) = \pi$ and $f_v = \pi/3$. The dashed line is for $\alpha = 4.77074$, the solid line for $\alpha = 1.0$ and the dotted line for $\alpha = -1.29324$.

d) When $|v| \geq 1$.

Noting that the case of $v > 1$ becomes identical to the case of $v < -1$ if we exchange f with $\pi - f$, let us focus ourselves on the case $v < -1$. We could regard the solutions in this case as some continuous deformation of the solutions in the case $|v| < 1$. Among the solitons discussed in the previous sections, those solutions with $f(r)$ lying between π and f_v will no longer exist.

When $N = 0$, the nontopological solitons without vortices exist only with $\pi > f(0) > 0$ and $f(\infty) = 0$. Both f, a are decreasing functions and $\alpha < -1$. When $f(0) = 0$ and $N \geq 1$, there are two kinds of solutions depending on whether $f(\infty) = \pi$ or 0 . For $f(\infty) = \pi$, they are topological lumps. The function $f(r)$ is increasing and $a(r)$ is decreasing to α . It should be that $\alpha > 1/2 - N$ if $v = -1$ and that $\alpha > 1 - N$ if $v < -1$. For $f(\infty) = 0$, they are nontopological solitons with vortices in the middle. The function $f(r)$ is increasing and then decreasing and $a(r)$ is a decreasing function with $\alpha < -(1 + N)$. When $f(0) = \pi$ and $N \leq -1$, there are topological lumps with $f(\infty) = 0$. Both $f(r)$ and $a(r)$ are decreasing functions and $\alpha < 0$.

We have seen that the various kinds of solitons in the symmetric and asymmetric phases. In the symmetric phase there exist nontopological solitons with or without vortices as well as

topological ones whose degrees are nonzero. The magnetic flux of solitons are not in general quantized. The mass and size of the solitons depend on their magnetic fluxes except when $v = \pm 1$ and $f(\infty) = v$. They in general carry the nonzero fractional angular momenta. In the asymmetric phase there can be a topological vortex whose degree takes a fractional value. Its magnetic flux is quantized and its mass is proportional to its vorticity.

The topological vortices in the broken phase with $|v| < 1$ can be classified into two classes for a given positive T , depending on whether $f(0) = 0$ or π . Their magnetic fluxes have opposite signs, but their angular momenta are along the same direction. We can imagine a configuration containing these two different vortices, which may be separated apart. As their topological charges T add up, their energy is bounded by the sum of their masses. Interesting question is then whether there is any self-dual configuration which represents this composite state. Such a self-dual configuration, if exists, cannot have rotationally symmetric configuration because the points where $f = 0$ and the points $f = \pi$ cannot be arranged to be rotationally symmetric.

We can extend our work in many directions. Clearly, we can introduce the parameter v in the pure Maxwell system, where we expect two types of vortices in the broken phase as in the pure Chern-Simons. There also can be topological lumps which are rotationally asymmetric. The statistical phases of vortices with fractional spins were explained by including the phase due to the Magnus force [9]. When there is a uniform external charge, due to the Magnus force single vortex behaves like a charged particle in a uniform magnetic field. It would be interesting to find out how those two types of vortices interacting with each other. The phase structure of the Chern-Simons Higgs system with a uniform background charge was found to be rich and interesting [10]. Our non-linear $O(3)$ models with the background charge will have even richer structures and deserve further investigation.

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